

GCSE to New A Level Transition in Mathematics

Coordinate geometry

1.

<p>9 (i)</p>	<p>Gradient = 4</p> $y - 7 = 4(x - 2)$ $y = 4x - 1$	<p>B1</p> <p>M1</p> <p>A1 3</p>	<p>Gradient of 4 so i</p> <p>Attempts equation of straight line through (2, 7) with any gradient</p>
<p>(ii)</p>	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $= \sqrt{(2 - 1)^2 + (7 - 2)^2}$ $= \sqrt{3^2 + 9^2}$ $= \sqrt{90}$ $= 3\sqrt{10}$	<p>M1</p> <p>A1</p> <p>A1 3</p>	<p>Use of correct formula for d or d^2 (3 values correctly substituted)</p> $\sqrt{3^2 + 9^2}$ <p>Correct simplified surd</p>
<p>(iii)</p>	<p>Gradient of AB = 3</p> <p>Gradient of perpendicular line = $-\frac{1}{3}$</p> <p>Midpoint of AB = $\left(\frac{1}{2}, \frac{5}{2}\right)$</p> $y - \frac{5}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$ $x + 3y - 8 = 0$	<p>B1</p> <p>B1 ft</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1 6</p> <p>12</p>	<p>SR Allow B1 for $-\frac{1}{4}$</p> <p>Attempts equation of straight line through their midpoint with any non-zero gradient</p> $y - \frac{5}{2} = \frac{-1}{3}\left(x - \frac{1}{2}\right)$ $x + 3y - 8 = 0$

2.

7(i)	Gradient = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$
(ii)	$y - 5 = -\frac{1}{2}(x - 6)$ $2y - 10 = -x + 6$ $x + 2y - 16 = 0$	M1 B1 ft	Equation of straight line through (6, 5) with any non-zero numerical gradient Uses gradient found in (i) in their equation of line
(iii)	EITHER $\frac{4-x}{2} = x^2 + x + 1$ $4 - x = 2x^2 + 2x + 2$ $2x^2 + 3x - 2 = 0$ $(2x - 1)(x + 2) = 0$ $x = \frac{1}{2}, x = -2$ $y = \frac{7}{4}, y = 3$ OR $y = (4 - 2y)^2 + (4 - 2y) + 1$ $y = 16 - 16y + 4y^2 + 4 - 2y + 1$ $0 = 21 - 19y + 4y^2$ $0 = (4y - 7)(y - 3)$ $y = \frac{7}{4}, y = 3$ $x = \frac{1}{2}, x = -2$	A1 3 *M1 DM1 A1 A1 4	Correct answer in correct form (integer coefficients) Substitute to find an equation in x (or y) Correct method to solve quadratic $x = \frac{1}{2}, x = -2$ $y = \frac{7}{4}, y = 3$ SR one correct (x,y) pair www B1
			8

3.

(i)	$\sqrt{(-1-4)^2 + (9-^{-}3)^2}$ =13	M1 A1 2	Correct method to find line length using Pythagoras' theorem cao
(ii)	$\left(\frac{4+^{-}1}{2}, \frac{^{-}3+9}{2}\right)$ $\left(\frac{3}{2}, 3\right)$	M1 A1 2	Correct method to find midpoint
(iii)	Gradient of AB = $-\frac{12}{5}$ $y - 3 = -\frac{12}{5}(x - 1)$ $12x + 5y - 27 = 0$	B1 M1 A1 A1 4	Correct equation for line, any gradient, through (1, 3) Correct equation in any form with gradient simplified $12x + 5y - 27 = 0$
			8

Expanding brackets

4.

$(4x^2 + 20x + 25) - (x^2 - 6x + 9)$ $= 3x^2 + 26x + 16$ <u>Alternative method using difference of two squares:</u> $(2x + 5 + (x - 3))(2x + 5 - (x - 3))$ $= (3x + 2)(x + 8)$ $= 3x^2 + 26x + 16$	M1	Square one bracket to give an expression of the form $ax^2 + bx + c$ ($a \neq 0, b \neq 0, c \neq 0$) One squared bracket fully correct
	A1	
	A1 3	All 3 terms of final answer correct
		M1 2 brackets with same terms but different signs A1 One bracket correctly simplified A1 All 3 terms of final answer correct
	3	

5.

(i)	$(3x+1)^2 - 2(2x-3)^2$ $= (9x^2 + 6x + 1) - 2(4x^2 - 12x + 9)$ $= x^2 + 30x - 17$	M1	Square to get at least one 3 or 4 term quadratic $9x^2 + 6x + 1$ or $4x^2 - 12x + 9$ soi $x^2 + 30x - 17$
		A1	
		A1 3	
(ii)	$2x^3 + 6x^3 + 4x^3 = 12x^3$ 12	B1	2 of $2x^3, 6x^3, 4x^3$ soi N.B. www for these terms , must be positive
		B1 2	12 or $12x^3$

6.

(i)	$(2x^2 - 5x - 3)(x + 4)$ $= 2x^3 + 8x^2 - 5x^2 - 20x - 3x - 12$ $= 2x^3 + 3x^2 - 23x - 12$	M1	Attempt to multiply a quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term) Expansion with no more than one incorrect term
		A1	
		A1 3	
(ii)	$2x^4 + 7x^4$ $= 9x^4$ 9	B1	$2x^4$ or $7x^4$ soi www $9x^4$ or 9
		B1 2	
		5	

Simultaneous equations (linear/nonlinear)

7.

$y = 2x - 4$		
$4x^2 + (2x - 4)^2 = 10$	M1*	Attempt to get an equation in 1 variable only
$8x^2 - 16x + 16 = 10$		
$8x^2 - 16x + 6 = 0$	A1	Obtain correct 3 term quadratic (aef)
$4x^2 - 8x + 3 = 0$		
$(2x - 1)(2x - 3) = 0$	M1dep*	Correct method to solve quadratic of form $ax^2 + bx + c = 0$ ($b \neq 0$) Correct factorisation oe
$x = \frac{1}{2}, x = \frac{3}{2}$	A1	Both x values correct
$y = -3, y = -1$	A1 A1 6	Both y values correct
		or
	6	one correct pair of values www B1 second correct pair of values B1

Completing the square

8.

$[(x - 6)^2 - 36] + 1$	B1	$(x - 6)^2$
$= (x - 6)^2 - 35$	M1	$q = 1 - (\text{their } p)^2$
	A1 3	$q = -35$
		3

9.

(i)	$\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{1}{4}$	B1	$a = \frac{5}{2}$
	$= \left(x - \frac{5}{2}\right)^2 - 6$	M1	$\frac{1}{4} - a^2$
		A1 3	cao

10.

3	$a(x + 3)^2 + c = 3x^2 + bx + 10$	B1	$a = 3$ soi
	$3(x^2 + 6x + 9) + c = 3x^2 + bx + 10$	B1	$b = 18$ soi
	$3x^2 + 18x + 27 + c = 3x^2 + bx + 10$	M1	$c = 10 - 9a$ or $c = 10 - \frac{b^2}{12}$
	$c = -17$	A1 4 4	$c = -17$

Inequalities

11.

3(i)	$3x - 15 \leq 24$ $3x \leq 39$ $x \leq 13$	M1	Attempt to simplify expression by multiplying out brackets
	or $x - 5 \leq 8$ $x \leq 13$	M1 A1	Attempt to simplify expression by dividing through by 3
(ii)	$5x^2 > 80$ $x^2 > 16$ $x > 4$ or $x < -4$	M1	Attempt to rearrange inequality or equation to combine the constant terms
		B1	$x > 4$
		A1	fully correct, not wrapped, not 'and'
			SR B1 for $x \geq 4$, $x \leq -4$
			5

12.

(i)	$-42 < 6x < -6$ $-7 < x < -1$	M1	2 equations or inequalities both dealing with all 3 terms
		A1 A1	-7 and -1 seen oe $-7 < x < -1$ (or $x > -7$ and $x < -1$)
(ii)	$x^2 > 16$ $x > 4$ or $x < -4$	B1	± 4 oe seen
		B1	$x > 4$
		B1	$x < -4$ not wrapped, not 'and'
		3 <u>6</u>	

Transformations of graphs

13.

(i)	$y = (x - 2)^2$	M1	$y = (x \pm 2)^2$
		A1	
		<u>2</u>	
(ii)	$y = -(x^3 - 4)$	B1	oe
		<u>1</u>	

14.

(i)	(a) Fig 3	B1	
	(b) Fig 1	B1	
	(c) Fig 4	B1	3
(ii)	$-(x - 3)^2$	M1	Quadratic expression with correct x^2 term and correct y -intercept and/or roots for their unmatched diagram (e.g. negative quadratic with y -intercept of -9 or root of 3 for Fig 2)
	$y = -(x - 3)^2$	A1	Completely correct equation for Fig 2
		<u>2</u> <u>5</u>	

Indices and surds

15.

(i)	$x^{\frac{1}{3}} = 2$ $x = 8$	B1	1	8	(allow embedded values throughout question 1)
(ii)	$10^t = 1$ $t = 0$	B1	1	0	
(iii)	$(y^{-2})^2 = \frac{1}{81}$ $y^{-4} = \frac{1}{81}$ $y = \pm 3$	B1 B1	2	$y = 3$ $y = -3$	

16.

2(i)	1	B1	1	
(ii)	$\frac{1}{2} \times 2^4$ $= 8$	M1 M1 A1	3 4	$2^{-1} = \frac{1}{2}$ or $32^{\frac{1}{5}} = 2$ or $2^5 = 32$ soi $32^{\frac{4}{5}} = 2^4$ or 16 seen or implied 8

17.

(i)	$3\sqrt{20}$ or $3\sqrt{2} \sqrt{5} \times \sqrt{2}$ or $\sqrt{180}$ or $\sqrt{90} \times \sqrt{2}$ $= 6\sqrt{5}$	M1 A1	2	Correctly simplified answer
(ii)	$10\sqrt{5} + 5\sqrt{5}$ $= 15\sqrt{5}$	M1 B1 A1	3	Attempt to change both surds to $\sqrt{5}$ One part correct and fully simplified cao
5				

18.

4(i)	$p = -1$	B1	1	$p = -1$
(ii)	$\sqrt{25k^2} = 15$ $25k^2 = 225$ $k^2 = 9$ $k = \pm 3$	M1 A1 A1	3	Attempt to square 15 or attempt to square root $25k^2$ $k = 3$ $k = -3$
(iii)	$\sqrt[3]{t} = 2$ $t = 8$	M1 A1	2	$\frac{1}{t^{\frac{1}{3}}} = \frac{1}{2}$ or $t^{\frac{1}{3}} = 2$ soi $t = 8$
6				

19.

(i) $n = -2$	B1 1
(ii) $n = 3$	B1 1
(iii) $n = \frac{3}{2}$	M1 $\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $(4^3)^{\frac{1}{2}}$ or $4 \times \sqrt{4}$ with brackets correct if used A1 2

20.

(i) $\sqrt{2 \times 100} = 10\sqrt{2}$	B1 1
(ii) $\frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$	B1 1
(iii) $10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$	M1 Attempt to express $5\sqrt{8}$ in terms of $\sqrt{2}$ A1 2

Sine/cosine rule and area of a triangle

21.

(i) $\cos \theta = \frac{6.4^2 + 7.0^2 - 11.3^2}{2 \times 6.4 \times 7.0}$ $= -0.4211$ $\theta = 115^\circ$ or 2.01 rads	M1 Attempt use of cosine rule (any angle) A1 Obtain one of 115° , 34.2° , 30.9° , 2.01, 0.597, 0.539 A1 3 Obtain 115° or 2.01 rads, or better
(ii) area = $\frac{1}{2} \times 7 \times 6.4 \times \sin 115$ $= 20.3 \text{ cm}^2$	M1 Attempt triangle area using $(\frac{1}{2})ab\sin C$, or equiv A1 2 Obtain 20.3 (cao)
5	

22.

(i) $\frac{TA}{\sin 107} = \frac{50}{\sin 3}$ $TA = 914 \text{ m}$	M1 Attempt use of correct sine rule to find TA , or equiv A1 2 Obtain 914, or better
(ii) $TC = \sqrt{914^2 + 150^2 - 2 \times 914 \times 150 \times \cos 70}$ $= 874 \text{ m}$	M1 Attempt use of correct cosine rule, or equiv, to find TC A1√ Correct unsimplified expression for TC , following their (i) A1 3 Obtain 874, or better
(iii) dist from $A = 914 \times \cos 70 = 313 \text{ m}$ beyond C , hence 874 m is shortest dist <i>OR</i> perp dist = $914 \times \sin 70 = 859 \text{ m}$	M1 Attempt to locate point of closest approach A1 2 Convincing argument that the point is beyond C , or obtain 859, or better SR B1 for 874 stated with no method shown

23.

(i) $\frac{BD}{\sin 62} = \frac{16}{\sin 50}$ $BD = 18.4 \text{ cm}$	M1	Attempt to use correct sine rule in $\triangle BCD$, or equiv.
	A1 2	Obtain 18.4 cm
(ii) $18.4^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos \theta$ $\cos \theta = 0.3998$ $\theta = 66.4^\circ$	M1	Attempt to use correct cosine rule in $\triangle ABD$
	M1	Attempt to rearrange equation to find $\cos BAD$ (from $a^2 = b^2 + c^2 \pm (2)bc \cos A$)
	A1 3	Obtain 66.4°
		5

Probability

24.

7i	$\frac{2}{9}$ or $\frac{7}{9}$ oe seen $\frac{3}{9}$ or $\frac{6}{9}$ oe seen $\frac{1}{8}$ or $\frac{7}{8}$ oe seen Correct structure All correct	B1 B1 B1 B1 B1	ie 8 correct branches only, ignore probs & values including probs and values, but headings not req'd
ii	$\frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9} + \frac{7}{10} \times \frac{6}{9}$ $\frac{14}{15}$ or 0.933 oe	M2 A1 3	or $\frac{3}{10} \times \frac{7}{9} + \frac{7}{10}$ or $1 - \frac{3}{10} \times \frac{2}{9}$ M1: one correct prod or any prod + $\frac{7}{10}$ or $\frac{3}{10} \times \frac{2}{9}$
iii	$\frac{3}{10} \times \frac{2}{9} \times \frac{7}{8} + \frac{7}{10} \times \frac{6}{9}$ $\frac{21}{40}$ or 0.525 oe	M2 A1 3	M1: one correct prod cao
	No ft from diag except: with replacement:	(i) structure: B1 (ii) $\frac{91}{100}$: B2 (iii) 0.553: B2	
Total		11	

25.

8i	$\sqrt{0.04} (= 0.2)$ $(1 - \text{their } \sqrt{0.04})^2$ $= 0.64$	M1 M1 A1 3	
ii	$1 - p$ seen M1 for either $2p(1 - p) = 0.42$ or $p(1 - p) = 0.21$ oe $2p^2 - 2p + 0.42 (= 0)$ or $p^2 - p + 0.21 (= 0)$ $\frac{2 \pm \sqrt{((-2)^2 - 4 \times 0.42)}}{2 \times 2}$ or $\frac{1 \pm \sqrt{((-1)^2 - 4 \times 0.21)}}{2 \times 1}$ or $(p - 0.7)(p - 0.3) = 0$ or $(10p - 7)(10p - 3) = 0$ $p = 0.7$ or 0.3	B1 M1 M1 M1 A1 5	$2pq = 0.42$ or $pq = 0.21$ Allow $pq = 0.42$ or opp signs, correct terms any order (= 0) oe Correct Dep B1M1M1 Any corr subst'n or fact'n Omit 2 in 2 nd line: max B1M1M0M0A0 One corr ans with no or inadeq wking: SC1 eg $0.6 \times 0.7 = 0.42 \Rightarrow p = 0.7$ or 0.6 $p^2 + 2pq + q^2 = 1$ B1 $p^2 + q^2 = 0.58$ } $p = 0.21/q$ } $p^4 - 0.58p^2 + 0.0441 = 0$ M1 corr subst'n or fact'n M1 $1 - p$ seen B1 $2p(1 - p) = 0.42$ or $p(1 - p) = 0.21$ M1 $p^2 - p = -0.21$ $p^2 - p + 0.25 = -0.21 + 0.25$ oe } M1 OR $(p - 0.5)^2 - 0.25 = -0.21$ oe } $(p - 0.5)^2 = 0.04$ M1 $(p - 0.5) = \pm 0.02$ $p = 0.3$ or 0.7 A1
Total		8	

26.

5(i)	$\frac{12}{22} \times \frac{11}{21}$ = $\frac{2}{7}$ oe or 0.286 (3 sfs)	M1 A1 2	or ${}^{12}C_2 / {}^{22}C_2$
(ii)	$\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13}$ or $\frac{8}{65}$ oe $\times \frac{3}{65}$ oe = $\frac{24}{65}$ or 0.369 (3 sfs)	M1 M1 A1 3	Numerators any order ${}^6C_2 \times {}^8C_1$:M1 3 x prod any 3 probs (any C or P) ${}^{15}C_3$:M1 (dep <1) $1 - (\frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} + 3 \times \frac{8}{15} \times \frac{7}{14} \times \frac{7}{13} + \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13})$: M2 one prod omitted or wrong: M1
(iii)	$\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15}$ oe $x^2 - x - 132 = 0$ or $x(x-1) = 132$ $(x-12)(x+11) = 0$ or $x = \frac{1 \pm \sqrt{(1^2 - 4 \times (-132))}}{2}$ No. of Ys = 12	M1 A1 M1 A1 4	not $\frac{x}{45} \times \frac{x}{44} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x}{45} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x-1}{45} = \frac{1}{15}$ oe fit 3-term QE for M1 condone signs interchanged allow one sign error Not $x = 12$ or -11 ans 12 from less wking, eg $12 \times 11 = 132$ or T & I: full mks Some incorrect methods: $\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15}$ oe M1 $x^2 + x = 132$ A0 $x = 11$ M1A0 $12 \times 11 = 132$ M1A1M1 $x = 12$ and (or "or") 11 A0 NB 12 from eg 12.3 rounded, check method
Total		9	

27.

(i)	$\frac{6}{14} \times \frac{5}{13} \times \frac{3}{12}$ $\times 3!$ oe = $\frac{45}{182}$ or 0.247 (3 sfs)oe	M1 M1 A1 3	${}^6C_1 \times {}^5C_1 \times {}^3C_1$ $\div {}^{14}C_3$ With repl M0M1A0
(ii)	$\frac{6}{14} \times \frac{5}{13} \times \frac{4}{12} + \frac{5}{14} \times \frac{4}{13} \times \frac{3}{12} + \frac{3}{14} \times \frac{2}{13} \times \frac{1}{12}$ = $\frac{31}{364}$ or 0.0852 (3 sf)	M2 A1 3	${}^6C_3 + {}^5C_3 + {}^3C_3$ M1 for any one ($\div {}^{14}C_3$)M1 all 9 numerators correct. With repl M1($6/14$) ³ +($5/14$) ³ +($3/14$) ³
Total		[6]	

Statistics

28.

i	$\frac{25}{10}$ = 2.5	M1 A1 2	Allow $\frac{25}{(9to10)}$ or 2.78: M1
ii	(19.5, 25) (9.5, 0)	B1 B1 2	Allow (24.5, 47) Both reversed: SC B1 If three given, ignore (24.5, 47)
iii	Don't know exact or specific values of x (or min or max or quartiles or median or whiskers). oe Can only estimate (min or max or quartiles or median or whiskers) oe Can't work out (.....) oe Data is grouped oe	B1 1	Exact data not known Allow because data is rounded

29.

(i) Read at 300 or 300.25 and 900 or 900.75 44.5 to 45.5 and 69 to 69.9 IQR 23.5 to 25.4	M1 A1 A1	3	or 44-46 and 68-70 incl. dep A1 Must look back, see method. No wking. ans in range: M1A1A1
(ii) 0.6 or 60% CF 720 63 to 64	M1 M1 A1	3	Seen or implied Seen or implied 55.5 to 56: SC B1
(iii) 1200 – 860 = 340	M1 A1	2	Allow 1200 – (850 to 890) 310 to 350
(iv) 340/1200 0.283^5 = 0.00183	M1 M1dep A1	3	their (iii)/1200 [their (iii)/1200] ⁵ exactly Allow 0.00114 to 0.00212 ≥ 2 sfs ${}^{340}C_5 / {}^{1200}C_5$ M1
(v) Incorrect reason or ambiguity: B0B0. Otherwise: Too low, or should be 26 or 27 or 2 or 3 higher	B2	2	eg IQR = 55–35 = 20 or IQR = value >27 or new info' implies straight line: B1 or originally, majority in range 35 – 55 are at top of this range: B1
		13	

30.

6(i)(a)	256	B1	1	(i)(b) & (ii)(abc): ISW ie if correct seen, ignore extras
(b)	Total unknown or totals poss diff or Y13 may be smaller or similar or size of pie chart may differ	B1	1	pie chart shows only proportions oe or no. of students per degree may differ not "no. of F may be less" not "Y13 may be larger"
(ii)(a)	B&W does not show frequencies oe	B1	1	or B&W shows spread or shows mks or M lger range
(b)	F generally higher or median higher F higher on average or F better mks F IQR is above M IQR F more compact M wide(r) range or gter IQR or gter variation or gter variance or more spread or less consistent M evenly spread or F skewed	B1		1 mk about overall standard, based on median or on F's IQR being "higher" 1 mk about spread (or range or IQR) or about skewness. must be overall, not indiv mks must be comparison, not just figures Examples: not F higher mean not M have hiest and lowest mks
		B1	2	condone F +ve skew

(c)	<u>Advantage:</u> B&W shows med or Qs or IQR or range or hiest & lowest or key values <u>Disadvantage:</u> B&W loses info' B&W shows less info' B&W not show freqs B&W not show mode B&W: outlier can give false impression hist shows more info hist shows freqs or fds hist shows modal class (allow mode) hist shows distribution better can calc mean from hist	B1	not B&W shows skewness not B&W shows info at a glance not B&W easier to compare data sets not B&W shows mean not B&W shows spread not B&W easier to calculate or easier to read not B&W does not give indiv (or raw) data not B&W does not show mean not hist shows freq for each mark not hist shows all the results not hist shows total
(iii)	$102 \times 51 + 26 \times 59$ $\div 128$ $= 52.6$ (3 sfs)	B1 2 M1 M1dep A1 3	allow adv of hist as disadv of B&W or 5202 + 1534 or 6736
Total		10	