MODULE 2

# Foundations of physics

CHAPTER 2.1

# PHYSICAL QUANTITIES AND UNITS

## Introduction

Physicists, engineers and other scientists around the world need to communicate their ideas and findings with one another in a consistent way that they will all understand. For this to happen effectively, there needs to be a common language within the scientific community. A kilogram, newton, joule or ampere in the mind and calculations of a British scientist must mean exactly the same thing to the Japanese or American physicist working on the other side of the world. Physical quantities make sense when they have a numerical value and a unit that can be understood by all of the scientific community. In this unit you will encounter the base SI units as well as the derived units of SI base units. You will become more familiar with the prefixes used to show multiples and submultiples of units. You will check the homogeneity of physical equations using SI units and you will deploy the conventions used for correctly labelling graph axes and table columns.

### All the maths you need

- Units of measurement
- Addition and subtraction of quantities
- Multiplication and division of quantities including prefixes and the use of standard form
- The conversion between units with different prefixes (*e.g.* $cm^3$ *and* $m^3$)

### What have I studied before?

- Common quantities and SI units
- Prefixes such as milli-, centi-, kilo-, mega- and giga-
- How to label graph axes and table columns

### What will I study in this chapter?

- That physical quantities have a numerical value and a unit
- Estimation of a range of physical quantities
- SI units including base units and derived units of SI base units
- Checking the homogeneity of physical equations using base units
- Prefixes and their symbols to indicate multiples and sub-multiples of units
- The conventions used for labelling graph axes and table columns

### What will I study later?

- The nature of units used to describe scalar and vector quantities (AS)
- The addition, subtraction, multiplication and division of scalar and vector units and quantities (AS)
- The magnitude and direction of vector quantities acting at an angle to the direction of application of a force (AS)
- Compound variables, e.g. density, pressure, momentum, Young modulus and kinetic energy (AS)
- The units and quantities associated with the areas of forces, motion, work and energy, materials, electricity, waves and quantum physics (AS)
- Converting between units for energy, e.g. from *eV* to J (AS)
- The quantities and units relating to circular motion and oscillations, including the radian (AL)
- The quantities and units used in the fields of capacitance, cosmology, fields and medical physics. (AL)

# 1 Physical quantities and units

**By the end of this topic, you should be able to demonstrate and apply your knowledge and understanding of:**

* physical quantities have a numerical value and a unit
* Système International (SI) base quantities and their units
* derived units of SI base units
* prefixes and their symbols to indicate decimal submultiples or multiples of units
* checking the homogeneity of physical equations using SI base units

## Physical quantities

In physics, a *quantity* is a measurement of something. Any quantity will have a numerical value indicating its size and an appropriate unit.

For example, there are many units that can be used to measure length. If we wanted to state the distance from Newcastle to Dubai, we would use kilometres, for example:

distance from Newcastle to Dubai = 5642 km.

If we wanted to state this distance in cm, then we would convert it from km to cm by multiplying by 100 000 and the distance would be stated as 564 200 000 cm. However, we usually state longer distances in km and shorter distances in m, cm, mm or even smaller units that are fit for purpose.

Some examples of physical quantities and units are shown in Table 1.

| Quantity | Example | | | |
|---|---|---|---|---|
| length | length of an aircraft | = | 36 metres | m |
| area | area of a football pitch | = | 6000 square metres | $m^2$ |
| volume | volume of a house | = | 900 cubic metres | $m^3$ |
| time | time for a TV advert | = | 52 seconds | s |
| mass | mass of a baby | = | 3.8 kilograms | kg |
| power | the power of a domestic light bulb | = | 60 watts | W |
| speed | speed of a lizard | = | 18 metres per second | m $s^{-1}$ |
| energy | the energy content of a chocolate bar | = | 4000 joules | J |
| current | the current passing through an electric oven | = | 30 amps | A |
| voltage | the mains voltage supplied to homes in the UK | = | 230 volts | V |

**Table 1** Examples of physical quantities and units.

## Units

We can express quantities in many different units. For example, we could refer to the length of a field in metres, yards, inches, furlongs, miles, kilometres or even cubits. However, this can lead to confusion when we need to convert from one unit to another. In 1960, the international scientific community agreed to adopt a single unit for each quantity. These are known as Système International (SI) units. The seven fundamental units are shown in Table 2. The units for every other scientific quantity can be derived from these fundamental units. For example, momentum is measured in kg m $s^{-1}$ and the units for density in kg m $s^{-3}$.

**LEARNING TIP**

You may be able to work out what formula to use by looking at the units of the quantity you want to calculate. For example, momentum is measured in kg m $s^{-1}$, so to find the momentum of an object you need to multiply mass (kg) by velocity (m $s^{-1}$).





| Quantity | Unit | Abbreviation |
|---|---|---|
| mass | kilogram | kg |
| length | metre | m |
| time | second | s |
| temperature | kelvin | K |
| electrical current | ampere | A |
| amount of substance | mole | mol |
| luminous intensity | candela | cd |

**Table 2** The seven fundamental quantities and their associated units and abbreviations.

## Unit prefixes

There is huge variation in the size of things. For example, the diameter of the Universe is about $10^{24}$ m, which is enormous. Conversely, the diameter of an atomic nucleus is incredibly small at about $10^{-15}$ m. We often use *standard form* to help us write down very big and very small numbers in a quick and efficient way, rather than typing very long numbers with lots of digits into calculators or writing them out in full when performing written calculations. We can also use *prefixes* to show the size of a quantity in comparison to its SI unit. These prefixes are shown in Table 3 and you must learn them.

| Prefix | Abbreviation | Standard form |
|---|---|---|
| pico- | p | $10^{-12}$ |
| nano- | n | $10^{-9}$ |
| micro- | *m* | $10^{-6}$ |
| milli- | m | $10^{-3}$ |
| centi- | c | $10^{-2}$ |
| deci- | d | $10^{-1}$ |
| kilo- | k | $10^{3}$ |
| mega- | M | $10^{6}$ |
| giga- | G | $10^{9}$ |
| tera- | T | $10^{12}$ |

**Table 3** SI prefixes and their abbreviations.

**LEARNING TIP**

These prefixes are the cause of many lost exam marks! Every time you convert into the standard unit, just do a quick check. If you are converting mm into m and have calculated that 6.4 mm = 6400 m, checking will show that you have *multiplied* by 1000 instead of *dividing* by 1000 (i.e. 6.4 mm = 0.0064 m). Check your work at every stage in a calculation, not just at the end. Areas and volumes can be even trickier – for example, the area of a rectangle 3 cm × 5 cm is not 0.15 $m^2$. You need to change the lengths into metres *before* doing the multiplication: area = 3 cm × 5 cm = 0.03 m × 0.05 m = 0.0015 $m^2$.

## Questions

1. Write the following in terms of fundamental units, using standard form.
   For example: the wavelength of red light = 600 nm; 600 nm = 6.0 × $10^{-7}$ m.
   (a) A raindrop has a diameter of 0.1 mm.
   (b) The distance from Land's End to John O'Groats is 1000 km.
   (c) The mass of a person is 60 000 g. (Note that kilogram is the only fundamental unit that itself has a prefix.)
   (d) An X-ray has a wavelength of 0.46 nm.
   (e) One day.
   (f) One year.
   (g) The area of a sheet of A4 paper is 29.7 cm × 21.0 cm = 624 $cm^2$.
   (h) The volume of a sphere is $\frac{4pr^3}{3}$. What is the volume of a ball with a diameter of 4 mm?
   (i) An electric current of 400 *m*A.

2. What are the prefixes that are:
   (a) smaller than pico
   (b) bigger than tera?

3. A component in a microprocessor is 60 nm long. Write down this length in *m*m, mm, m and km using the powers of ten notation.

4. Copy Table 4 and complete it using the correct prefix and SI unit.

| Quantity | Calculation | Correct value, prefix and SI unit |
|---|---|---|
| Area of table = 85 cm × 120 cm | Area = 0.85 m × 1.20 m | Area = 1.02 $m^2$ |
| Length of a car = 4200 mm | | |
| Volume of a room = 1000 cm × 3400 cm × 85 000 mm | | |
| Resistance = $\frac{420\text{ kV}}{105\text{ mA}}$ | | |
| A speed of 20 km $h^{-1}$ | | |

**Table 4**

5. Do the following calculations and conversions without using a calculator:
   (a) Convert 4.6 × $10^{-3}$ m to mm.
   (b) Convert 780nm to m.
   (c) Express the area of a 50 cm by 80 cm table in $m^2$.
   (d) Find the prefix that is equal to $\frac{\text{kilo}}{\text{nano}} \times \text{micro} \times \frac{\text{mega}}{\text{tera}}$
   (e) A storage facility has the dimensions 30 m × 12 m × 18 m. How many biscuits measuring 45 mm × 3.2 cm × 80 mm could be fitted into the building?

# 2 Estimated physical quantities

**By the end of this topic, you should be able to demonstrate and apply your knowledge and understanding of:**

- making estimates of physical quantities

## What is estimation?

When we estimate a value, we use sensible and simple substitutes in order to perform a calculation – the sort that we can perform mentally if required. An estimation is an educated guess resulting in an answer that is close, but not exactly equal, to the true answer.

Suppose you were given the following numbers to add:

97, 105, 48, 980, 312, 8, 53, 202

You might try to add them up mentally but it would probably lead to an arithmetical error because there are too many digits to memorise. Instead, you could pick numbers that are accurate to 1 significant figure, leading to:

100, 100, 50, 1000, 300, 10, 50, 200

This would give an 'estimation value' of 1810 – the true value is 1805. This means that the estimation is better than 99% accurate!

### WORKED EXAMPLE 1

To calculate the resistance of an electrical component, we use the equation:

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}}$$

It is found that a potential difference of 24.5 V will produce a current of 0.95 A in a component (see Figure 1).

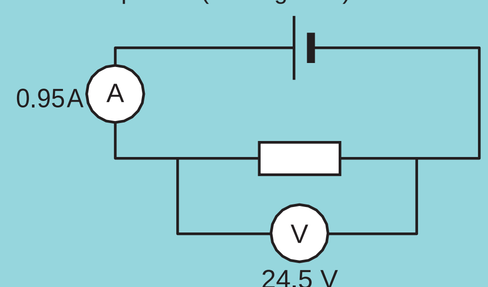



**Figure 1**

(a) What is the value of the potential difference to 2 significant figures?

(b) What is the value of the current to 1 significant figure?

(c) Use these values to estimate the resistance of the component.

**Answers**

(a) 24.5 V is stated to 3 significant figures. To round to 2 significant figures we look at the third digit and see that it is equal to 5, so we round up from 24.5 to 25 V.

(b) 0.95 A becomes 1 A to 1 significant figure.

(c) Resistance $= \frac{25\text{ V}}{1\text{ A}} = 25\ \Omega$

### WORKED EXAMPLE 2

The dimensions of a rectangular cuboid are 48.9 cm, 103.2 cm and 12.7 cm, as shown in Figure 2.

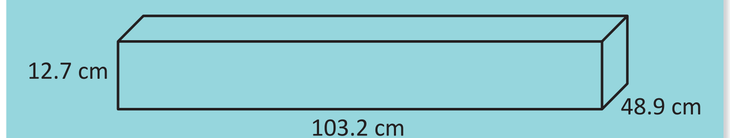



**Figure 2**

(a) Calculate the exact volume of the cuboid in $cm^3$ using $V = b \times l \times h$.

(b) Make an estimate for the volume of the cuboid in $cm^3$ by rounding each value to 1 significant figure.

(c) What is the percentage accuracy of the estimate compared with the true value?

**Answers**

(a) $V$ = 48.9 cm × 103.2 cm × 12.7 cm = 64 090.296 $cm^3$

(b) $V$ = 50 cm × 100 cm × 10 cm = 50 000 $cm^3$

(c) % accuracy $= \frac{50\,000}{(64\,090.296)} \times 100 = 78\%$

### LEARNING TIP

Many students muddle units when estimating and end up with unrealistic answers such as:

- very fast speeds for cars in kinematics questions, e.g. the car that travels at 5000 $m s^{-1}$
- the mass of an apple being 100 kg instead of 100 g.

Always double check your calculations or use a different method and see if you get a similar answer.

## Estimating using standard form

When estimating values using numbers in standard form, you have to be careful to follow some rules, which are explained in more detail in Topic 2.1.1. Your answer should be accurate to the correct 'order of magnitude' – that is, your answer should be within one power of ten of the correct answer.

### WORKED EXAMPLE 3

How many times heavier than a proton is a DNA molecule?

**Answer**

Using Table 1, the mass of a DNA molecule is $3 \times 10^{-18}$ kg and the mass of a proton is $1.6 \times 10^{-27}$ kg.

Using standard form, an estimation of the ratio of mass of DNA : mass of a proton is:

$10^{-18} : 10^{-27}$, so the DNA is $10^9$ times heavier.

The answer from the full calculation is $1.88 \times 10^9$, which is the same order of magnitude as the estimation.





| | Mass/kg | | Distance/m | | Time/s | | Power/W |
|---|---|---|---|---|---|---|---|
| galaxy | $10^{41}$ | | | | | | |
| Sun | $6 \times 10^{26}$ | | | | | Sun | $10^{26}$ |
| Earth | $6 \times 10^{24}$ | | | | | | |
| Moon | $2 \times 10^{24}$ | Universe<br>Earth to Sun | $10^{24}$<br>$1.5 \times 10^{11}$ | since Big Bang<br>age of Earth | $5 \times 10^{17}$<br>$2 \times 10^{17}$ | from Sun to Earth | $10^{17}$ |
| supertanker | $5 \times 10^{8}$ | Sun's radius<br>Earth to Moon<br>Earth's radius | $7 \times 10^{8}$<br>$4 \times 10^{8}$<br>$6.4 \times 10^{6}$ | human lifetime<br>year | $2.5 \times 10^{9}$<br>$3.2 \times 10^{7}$ | power station<br>locomotive | $10^{9}$<br>$5 \times 10^{6}$ |
| train | $6 \times 10^{5}$ | | | | | family car | $10^{5}$ |
| jumbo jet | $1.6 \times 10^{5}$ | London–Paris<br>marathon race | $3.4 \times 10^{5}$<br>$4.2 \times 10^{4}$ | day | 86 400 | | |
| lorry | 40 000 | radio wavelength | 1500 | hour<br>10 000 m race | 3600<br>2000 | | |
| car | 1000 | race track<br>Egyptian pyramid | 400<br>150 | time to boil egg | 300 | 1 horse power<br>human power | 746<br>100 |
| person | 80 | height of house | 10 | minute | 60 | light bulb | 30 |
| new baby | 3 | height of person | 2 | | | | |
| mouse | 0.1 | eye radius | 0.02 | heart beat | 1 | clock | $10^{-3}$ |
| human egg | $2 \times 10^{-6}$ | light wavelength<br>X-ray wavelength<br>H atom diameter | $5 \times 10^{-7}$<br>$10^{-10}$<br>$5 \times 10^{-11}$ | | | | |
| blood corpuscle | $1 \times 10^{-12}$ | proton diameter | $1 \times 10^{-15}$ | period of light wave | $2 \times 10^{-15}$ | | |
| DNA molecule | $3 \times 10^{-18}$ | | | | | | |
| uranium-238 atom | $4 \times 10^{-25}$ | | | | | | |
| proton | $1.6 \times 10^{-27}$ | | | | | | |
| electron | $9 \times 10^{-31}$ | | | | | | |

**Table 1** Orders of magnitude relating to mass, distance, time and power.

## Questions

Use Table 1 to estimate:

(a) how long it will take light to reach the Earth from the Sun

(b) how long it will take sound to travel from London to Paris

(c) how many stars there are in a galaxy

(d) how many protons you could fit into a uranium nucleus

(e) how many cars you could run from a power station each second.

A car travels across Africa taking a total time of 3 weeks, during which it covers a distance of 5468 km. Estimate the car's speed in $m s^{-1}$.

We calculate the density of a material using the formula

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

An object will float in water if it has a density lower than 1 $g cm^{-3}$.

(a) Estimate the density of the cuboid shown in Figure 4.

(b) Will it float or sink when placed in water?

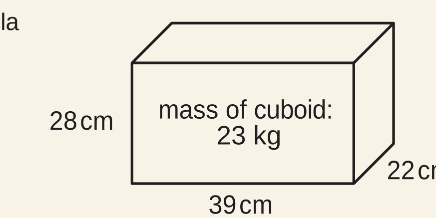



**Figure 3**

2.1

# 3 Systematic errors and random errors

**By the end of this topic, you should be able to demonstrate and apply your knowledge and understanding of:**

* systematic errors and random errors in measurement

## What is an error?

No matter how careful we try to be when planning and implementing an experimental or investigative procedure, there will always be occasions where errors occur in the collection of data and in the subsequent calculation process.

We come across the term ‘error’ frequently in everyday language, and some common definitions are:

- the condition of having incorrect or false knowledge
- an incorrect belief or judgement
- a mistake or wrongdoing
- the difference between a computed value or a measured value and a true or theoretically correct one.

The last of these definitions is possibly the best one to use when considering errors in science.

When conducting investigations it is important to minimise errors because they will lead to an incorrect conclusion, value or theory being produced. This will be a deviation from the scientific ‘truth’.

## Random errors

Random errors in experimental measurements are caused by unknown and unpredictable changes during the experiment. These may occur because of changes in the instruments or in the environment.

Humans can also make errors when reading or recording a value. A human error is often a ‘one-off’, caused by a simple mistake. This means that results will be inconsistent when repeated, so it is often easy to spot a human error because it is wildly different from the corresponding values from other repeats.

Random errors and human errors are not really the same. Random errors will always occur due to small changes in environmental conditions, such as small temperature changes or fluctuations in light levels. Human errors are just mistakes, such as when an experimenter misreads a value and records a reading of 2.84 A instead of the correct value of 2.48 A.

### Examples of random errors

- Variable heating in circuits causing variations in the current being measured.
- An unexpectedly large extension of a metal wire due to a fault in its structure.

**WORKED EXAMPLE 1**

A student was collecting data in order to determine the relationship between the length of a metal wire and its electrical resistance. He had planned the investigation well and used suitable and appropriate apparatus to collect the data he needed.

- He measured the length of the wire to the nearest millimetre, the current to 0.01 A and the voltage to 0.01 V.
- He recorded his average resistance values to 1 decimal place.

Table 1 shows his collected results.

(a) Can you identify possible random errors and human errors in his results table?
(b) What suggestions would you make to help the student to recognise these errors quickly?

**Answers**

(a) The current values for 0.200 m (20.0 cm) appear to have been written incorrectly (as 0.80 A instead of 0.08 A) on both occasions. The student may have misread the value, or their partner (if they had one) may have read or stated the value incorrectly. Either way, the figure is incorrect, leading to a value for the resistance that is ten times too small.

You could also argue that there is variation in some repeat readings for the voltage and current, which may have been caused by heating or by a power surge. For example, the potential difference readings for the 0.400 m length are 2.91 V and 2.87 V – close but not identical. This constitutes a random error, caused by unknown and unpredictable changes in the circuit conditions. Random errors tend to even out under statistical analysis.

(b) When the graph of resistance against length is plotted, it will be obvious that the 0.200 m reading is anomalous. However, it is better to spot such errors early, so that new readings can be taken using the same equipment, in the same conditions and at the same time as the other readings. Minor variations and fluctuations in the equipment and conditions can provide different values, even though this should not be the case theoretically.

You could suggest that the student looks for a pattern in the results that would be expected from the planning and hypothesis generation. For this investigation, the hypothesis may have been that the resistance is directly proportional to the length of the wire, so reductions in resistance or increases in current from 0.100 m to 0.200 m should ring alarm bells and be checked. In addition, the student could take a third set of results (a second set of repeats), although this is not always possible, or even necessary if the results are close.





| Length/cm | Potential difference 1/V | Current 1/A | Potential difference 2/V | Current 2/A | Average resistance/Ω |
|---|---|---|---|---|---|
| 10.0 | 1.10 | 0.09 | 1.00 | 0.09 | 11.7 |
| 20.0 | 1.86 | 0.80 | 1.90 | 0.80 | 2.3 |
| 30.0 | 2.74 | 0.07 | 2.74 | 0.08 | 34.2 |
| 40.0 | 2.91 | 0.06 | 2.87 | 0.06 | 48.2 |
| 50.0 | 2.92 | 0.05 | 2.92 | 0.05 | 58.3 |
| 60.0 | 3.52 | 0.05 | 3.52 | 0.05 | 70.3 |
| 70.0 | 3.83 | 0.04 | 3.83 | 0.04 | 95.8 |

**Table 1** A student’s results.

**KEY DEFINITION**

A **systematic error** is an error that does not happen by chance but instead is introduced by an inaccuracy in the apparatus or its use by the person conducting the investigation.

A **random error** is an error in an experiment caused by unknown or unpredictable changes to the apparatus or conditions.

## Systematic errors

Systematic errors in experimental observations usually involve the measuring instruments. They may occur because:

- there is something wrong with the instrument or its data-handling system
- the instrument is used wrongly by the student conducting the experiment.

Systematic errors are often present in all the readings taken and can be removed once identified. For example, if the needle on a set of scales is pointing to 25 g when nothing is on the pan, all values recorded will be bigger than the true mass by 25 g. To obtain the true value, simply subtract 25 g from each of the readings collected.

### Examples of systematic errors

- The scale printed on a metre rule is incorrect and the ruler scale is only 99.0 cm long.
- The needle on an ammeter points to 0.1 A when no current is flowing. Each value recorded will, therefore, be bigger than the true value by 0.1 A. This is an example of a ‘zero error’ – the apparatus shows a non-zero value when it should be registering a value of exactly zero.
- A thermometer has been incorrectly calibrated, so it constantly gives temperature readings that are 2 °C lower than the true temperature.

**WORKED EXAMPLE 2**

What changes would you make to the results that you collected if you were using an ammeter that looked like Figure 1 before it was connected?

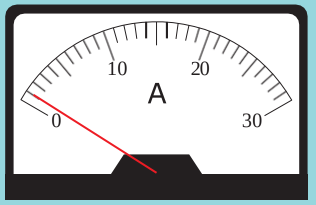



**Figure 1**

**Answers**

The ammeter has a zero error – it reads 1 A even when it is not in use. You need to reset the needle so that it points to 0 A when not in use or subtract 1 A from each value taken during the experiment.

## Question

1 Read the statements below and explain how each situation could be caused by a random error and/or a systematic error.

(a) ‘This voltmeter is registering a value of 0.1 V even though it is not connected in the circuit!’
(b) ‘This micrometer points to 0.001 mm even when it is supposed to read exactly 0.’
(c) ‘My current readings for the length of 40 cm were 1.56 A, 1.54 A and 1.58 A.’
(d) ‘The wind was blowing and causing the temperature of my water bath to decrease.’
(e) ‘The stopwatch readings are all different when I time the period of oscillation of a pendulum.’

# 4 Precision and accuracy

**By the end of this topic, you should be able to demonstrate and apply your knowledge and understanding of:**

- the terms precision and accuracy

**KEY DEFINITION**

**Precision** is the degree to which repeated values, collected under the same conditions in an experiment, show the same results.

**Accuracy** is the degree to which a value obtained by an experiment is close to the actual or true value.

## Introduction

The terms 'accuracy' and 'precision', are often confused or taken to mean the same thing. In everyday conversation this is not really a problem but, in the study of physics, it is essential that you understand and use these terms correctly.

At a simple level, the difference between accuracy and precision can be thought of using the sport of archery. Arrows are fired towards a target, with the intention of scoring as many points as possible. To get the maximum score, the archer has to hit the gold circle at the centre of the target – this is worth 10 points. Hitting the target further away from the central point means that the archer scores fewer and fewer points.


**Figure 1** The closer to the centre of the target an arrow lands, the more accurate it is. The closer the arrows cluster together, the more precise the shooting is. The aim is to be accurate *and* precise.

When the shots are accurate and precise, they all land in the middle of the target. When the shots are precise, they do not all land in the centre of the target, but they land very close to one another. Low accuracy and low precision can be thought of as arrows landing far away from the centre of the target and far away from each other.

## Accuracy and precision defined

An experiment is *accurate* if the quantity being measured has a value that is very close to the commonly accepted or true value. For example, if you were carrying out an experiment to determine the value of the acceleration due to gravity, then a value of $9.83\,m\,s^{-2}$ would be more accurate than one of $9.17\,m\,s^{-2}$. This is because the commonly accepted value of the acceleration due to gravity, $g$, is $9.81\,m\,s^{-2}$.

The term *precision* relates to how close together repeat values are. The smaller the spread, or range, of the repeat values (the closer they are to each other) the higher the precision. For example, if you conduct an investigation using instruments that enable you to determine the charge on an electron to ±0.1%, that is much better than obtaining a value that could be out by ±5%.

Precision is often used in conjunction with the term 'resolution', which relates to the smallest change in a quantity that an instrument can measure. An instrument that can measure a change of ±0.01 has a better resolution than one that can measure a change of ±0.1. This will lead to higher precision in the repeat readings.





**WORKED EXAMPLE**

Suggest a value for the acceleration due to gravity, $g$, that is:

(a) accurate and precise
(b) accurate but imprecise
(c) inaccurate but precise.

**Answers**

(a) An accurate and precise value for $g$ could be $9.80 \pm 0.02\,m\,s^{-2}$.
(b) An accurate and imprecise value for $g$ could be $9.80 \pm 1.50\,m\,s^{-2}$.
(c) An inaccurate but precise value for $g$ could be $8.80 \pm 0.05\,m\,s^{-2}$.

Systematic errors tend to have the same value throughout an investigation. For example, a thermometer that has been incorrectly calibrated may give temperature values that are incorrect by 1.0 °C each time – for example, the temperature is indicated to be 89 °C when in fact it is only 88 °C. To obtain the 'true' value, you would subtract the systematic error of 1 °C from every reading.

So how does a systematic error affect accuracy and precision? Accuracy will be affected by a systematic error that is not accounted for, as the systematic error takes the measured value away from its true value. However, a systematic error will not affect the precision, which is ultimately determined by the resolution of the apparatus being used. If a thermometer can read a temperature to the nearest 0.5 °C, then this precision will not be affected by a systematic error.

Random errors occur due to unknown and unpredictable factors such as wind variation, temperature changes, power surges and background radiation changes. Due to their random nature, random errors can produce variations in the measured value that are both above and below the 'true' value. The effect of random variation can be reduced by taking repeated measurements and reporting the mean.

## Questions

1. An investigation was conducted to determine the speed of sound in air at a given temperature. The accepted value for sound in air at this temperature is $343.4\,m\,s^{-1}$. Comment on the accuracy and precision of the values recorded below:

   $343.6\,m\,s^{-1}$, $344.1\,m\,s^{-1}$, $343.2\,m\,s^{-1}$, $343.1\,m\,s^{-1}$, $342.9\,m\,s^{-1}$, $343.7\,m\,s^{-1}$

2. The same investigation as described in question 1 was repeated by two students exactly one week later in the same room. Provide another set of six possible values they could have obtained that would be:
   (a) less accurate but more precise
   (b) more accurate and more precise
   (c) more accurate but less precise.

3. Find out how precise and accurate the following values are:
   (a) the density of gold is calculated as $21.4 \pm 0.04\,g\,cm^{-3}$
   (b) the speed of sound in copper is found to be $5050 \pm 800\,m\,s^{-1}$
   (c) the mass of the Earth is found to be $5.8 \times 10^{24}\,kg \pm 0.9\%$.

# 5 Absolute and percentage uncertainties

**By the end of this topic, you should be able to demonstrate and apply your knowledge and understanding of:**

* absolute and percentage uncertainties when data are combined by addition, subtraction, multiplication, division and raising to powers

**KEY DEFINITION**

**Absolute error** is the difference between a measured value and the actual value.

**Percentage error** is the difference between a measured and a true value expressed as a percentage.

## Introduction

In science, we conduct some investigations to determine the value of a physical quantity. For example, we may measure the length of a ramp using a metre ruler that has a centimetre scale and also a millimetre scale. When we make our measurements, we hope to get a value close to the true length of the ramp.

However, there is always uncertainty involved when we make measurements. We hope to get 'good' values but we have to accept that not every measurement we make will be the same every time we repeat it. Measurements may change because of the accuracy of the instrument we are using, the size of the smallest division on the scale or the skill of the person making the measurement. This means that when we consider the uncertainty in a particular measurement, we may need to estimate a value for it based on the following three factors.

## Absolute uncertainty

Imagine that you want to measure the diameter of a ball. You are using a metre ruler that has centimetre and millimetre graduations. You find that the diameter of the ball is 63 mm, so you could state that the diameter of the ball is 63 ± 1 mm.

When taking *single* readings, the *absolute uncertainty* is the smallest division on the measuring instrument used and the measurement should be stated as 'measured value ± precision of measuring instrument'.

Determining uncertainty in investigations that involve the use of a stopwatch can be problematic. Despite the fact that most stopwatches used in schools will give values accurate to 0.01 s, it is not possible for human reaction times to be this quick. Instead we use the uncertainty of the reaction time (about 0.5 seconds) when stating a time value and its absolute uncertainty.

**WORKED EXAMPLE 1**

A metre ruler is used to measure the height of a table and it is found to be 780 mm. How would you state this value if the ruler used had:

(a) a millimetre scale

(b) a centimetre scale?

**Answers**

(a) 780 ± 1 mm

(b) 78 ± 1 cm

## Percentage uncertainty of a single value

We often need to calculate a percentage uncertainty and include it in our evaluation. To calculate the percentage uncertainty of a single value, we use the equation:

$$\text{percentage uncertainty} = \left(\frac{\text{uncertainty}}{\text{measured value}}\right) \times 100\%$$

In this case, the uncertainty value is given by the precision of the instrument and the measured value is what has been read from the meter or ruler.

**LEARNING TIP**

Uncertainties should only really be stated to 1 significant figure, so the uncertainty in (a) should be quoted as ±1% and in (b) as 0.2%. This is because there is 'uncertainty in the uncertainty' – it is an estimation and we do not expect, or need, it to be too precise.

**WORKED EXAMPLE 2**

A digital ammeter is used to measure the current flowing in a series circuit and it is accurate to 0.01 A. What is the percentage uncertainty for a current of:

(a) 0.8 A

(b) 4.3 A?

**Answers**

(a) $\text{percentage uncertainty} = \left(\frac{0.01}{0.8}\right) \times 100\% = 1.25\%$

(b) $\text{percentage uncertainty} = \left(\frac{0.01}{4.3}\right) \times 100\% = 0.233\%$





## Percentage uncertainty for a number of repeat readings

When repeat readings are taken, random errors will be present. This may be due to fluctuations in temperature or unknown changes that we cannot account for or control.

Imagine that we have taken a set of potential difference readings, as shown in Table 1.

| Reading 1/V | Reading 2/V | Reading 3/V | Reading 4/V | Mean value/V |
|---|---|---|---|---|
| 3.89 | 3.88 | 3.86 | 3.90 | 3.88 |

**Table 1**

To calculate the percentage uncertainty, we use the following steps.

1. Write down the repeat readings in the table (as shown in Table 1).
2. Find and record the average of these readings (3.88).
3. Find the range of the repeat readings – this is the largest value minus the smallest value. In this case it is 3.90 V − 3.86 V = 0.04 V.
4. Halve the range to find the uncertainty value. In this case it is 0.02 V.
5. Divide the uncertainty value by the mean value and multiply by 100 to give the percentage uncertainty. In this case:

$$\frac{0.02\text{ V}}{3.88\text{ V}} \times 100\% = 0.5\%$$

Remember, we state the uncertainty to 1 significant figure.

**WORKED EXAMPLE 3**

The temperature of a room was measured several times and the values recorded in Table 2.

| Reading 1 (°C) | Reading 2 (°C) | Reading 3 (°C) | Reading 4 (°C) | Mean value (°C) |
|---|---|---|---|---|
| 21.4 | 21.3 | 21.4 | 21.1 | 21.3 |

**Table 2**

Calculate:

(a) the absolute uncertainty in the readings

(b) the percentage uncertainty in the readings.

**Answers**

(a) The absolute uncertainty in the repeat readings is given by half the range of the values, so:

range = maximum value − minimum value
= 21.4 °C − 21.1 °C = 0.3 °C

So the absolute uncertainty is 0.15 °C, or 0.2 °C to 1 significant figure. We would quote the temperature as (21.3 ± 0.2) °C.

(b) To find the percentage uncertainty, we divide the value of half the range by the mean value, and then multiply by 100:

$$\text{percentage uncertainty} = \left(\frac{0.15}{21.3}\right) \times 100\% = 0.7\%$$

## The rules for determining percentage uncertainties

At AS level you will be expected to determine the final percentage uncertainty in a compound quantity. This is based on the calculations that need to be carried out in order to find the quantity. The rules are:

- for a compound quantity of the form $y = ab$, the rule is:
  % uncertainty in $y$ = % uncertainty in $a$ + % uncertainty in $b$
- for a compound quantity of the form $y = \frac{a}{b}$, the rule is:
  % uncertainty in $y$ = % uncertainty in $a$ + % uncertainty in $b$
- for a compound quantity of the form $y = a^2$ the rule is:
  % uncertainty in $y$ = 2 × % uncertainty in $a$
- for a compound quantity of the form $y = a^n$, the rule is:
  % uncertainty in $y$ = $n$ × % uncertainty in $a$.

When you add or subtract readings, you must add together the absolute uncertainties of the individual readings to find the combined absolute uncertainty. For example, if you have three measurements – 78 ± 1 cm, 43 ± 1 cm and 57 ± 1 cm the total measurement is 178 ± 3 cm.

Alternatively, if you wish to subtract 42 ± 2 mm from 87 ± 2 mm, the final measurement is 45 ± 4 mm.

**WORKED EXAMPLE 4**

(a) Potential difference is calculated by using the equation $V = IR$. If the % uncertainty in the current, $I$, is 10% and the % uncertainty in the resistance, $R$, is 5% then what is the % uncertainty in the potential difference, $V$?

(b) A Young modulus is calculated using the equation Young modulus $= \frac{\text{stress}}{\text{strain}}$. If the percentage uncertainty in the stress is 8% and the % uncertainty in the strain is 6% then what is the % uncertainty in the value for the Young modulus?

(c) The density of a cube is calculated using density $= \frac{\text{mass}}{\text{volume}}$. If the % uncertainty in the mass is 1% and the % uncertainty of the length of one of the sides of the cube is 2% then what will be the percentage uncertainty in the density of the cube?

**Answers**

(a) $V = IR$ has the form $y = ab$, so we add the individual % uncertainties in the current and the resistance to get a 15% uncertainty in $V$.

(b) The equation has the form $y = \frac{a}{b}$. This means that we add the values for the stress and strain uncertainties to give an uncertainty in the Young modulus of 14%.

(c) The % uncertainty in the mass is 1%.
Volume is calculated using $V = a^3$, so the % uncertainty in the volume will be 3 × the % uncertainty in the length, i.e. 6%.
Because density $= \frac{\text{mass}}{\text{volume}}$ and has the form $y = \frac{a}{b}$, we add the % uncertainty in the mass to the % uncertainty in the volume, giving a % uncertainty in the density of 7%.

## Questions

1. What is the percentage uncertainty in a length of 76 mm measured with a metre ruler that has a millimetre scale?
2. What is the percentage uncertainty for these values for the time of a pendulum swing?
   12.83 s, 12.87 s, 12.85 s, 12.81 s
3. A compound variable is calculated using the formula $y = \frac{ab}{c^3}$.
   What is the percentage uncertainty in $y$ if the percentage uncertainty in $a$ is 3%, $b$ is 6% and $c$ is 2%?

# 6 Graphical treatment of errors and uncertainties

**By the end of this topic, you should be able to demonstrate and apply your knowledge and understanding of:**

* graphical treatment of errors and uncertainties; line of best fit; worst line; absolute and percentage uncertainties; percentage difference

## Using graphs

We make full use of graphs in physics to show relationships between pairs of dependent variables and independent variables. Graphs are a useful, highly visual way of demonstrating the relationship between two variables, showing patterns and trends and allowing us to determine values from measurements of the gradient and the $y$-intercept.

Graphs are most effective when:

- the scale of the graph has been drawn to cover as much of the graph paper as possible
- the points are plotted clearly
- the lines of best fit and worst fit are drawn clearly
- the gradient can be calculated easily using the extreme values on the $x$-axis and the $y$-axis
- the $y$-intercept can be read clearly and accurately using the scale on the $y$-axis.

**KEY DEFINITION**

**Percentage difference** is the difference between two values, divided by the average and shown as a percentage.

## Determining the uncertainty in the gradient from the maximum and minimum gradients

It is possible to determine the uncertainty in a gradient by drawing lines of maximum and minimum gradient through the appropriate points on the graph. If there is only a small amount of scatter then error bars can be incorporated into the graph to help this to happen.

The uncertainty in a gradient can be determined as follows:

1. Add error bars to each point if they are not scattered much and lie near to the line of best fit.
2. Draw a line of best fit through the scattered points or through the error bars. The line of best fit should go through as many points as possible, with equal numbers of points above and below the line. Discard any major outliers.
3. Calculate the gradient of the line of best fit.
4. Do the same for the worst fit line, which may be more steep or less steep than the line of best fit.
5. To find the uncertainty from the graph, work out the difference between the gradients of the line of best fit and the line of worst fit. This should be expressed as a positive value (the modulus). The equation you use is:

   uncertainty = (gradient of best fit line) − (gradient of worst fit line)
6. Calculate the percentage uncertainty in the gradient using the following equations:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{(\text{gradient of best fit line})} \times 100\%$$

Alternatively, you can draw a graph that has a line of best fit, a maximum gradient line and a minimum gradient line. In this case, the uncertainty is half the difference between the maximum and minimum gradients, as shown in Worked example 1.





**WORKED EXAMPLE 1**

An experiment is performed to determine the relationship between the current flowing through an electrical component and the potential difference across it. The current is measured to ±2.5 mA and the potential difference to ±0.1 V; these are the values used to plot the error bars on the $y$- and $x$-axes respectively.

Calculate:

(a) the gradient of the line of best fit
(b) the gradient of the minimum line
(c) the gradient of the maximum line
(d) the uncertainty in the gradient.

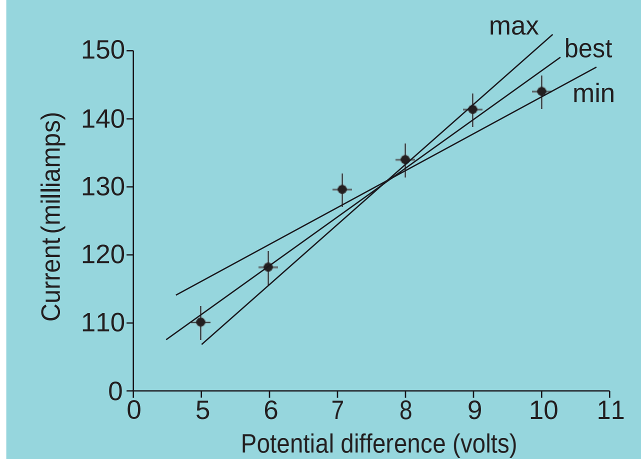



**Figure 1**

**Answers**

(a) $\text{Gradient} = \dfrac{147\,\text{mA} - 107\,\text{mA}}{10\,\text{V} - 4.5\,\text{V}} = 7.27\,\text{mA}\,\text{V}^{-1}$

(b) $\text{Gradient} = \dfrac{145\,\text{mA} - 115\,\text{mA}}{10.5\,\text{V} - 5.0\,\text{V}} = 5.45\,\text{mA}\,\text{V}^{-1}$

(c) $\text{Gradient} = \dfrac{152\,\text{mA} - 106\,\text{mA}}{10\,\text{V} - 5.0\,\text{V}} = 9.20\,\text{mA}\,\text{V}^{-1}$

(d) The uncertainty in the gradient is half the difference between the minimum line gradient and the maximum line gradient.

$$\text{uncertainty} = \tfrac{1}{2} \times (9.20 - 5.45) = 1.875\,\text{mA}\,\text{V}^{-1}$$

The gradient can now be written as $(7.3 \pm 2)\,\text{mA}\,\text{V}^{-1}$ with the gradient given to 2 significant figures and the uncertainty quoted to 1 significant figure.

## Determining the uncertainty in the $y$-intercept from the maximum and minimum gradients

You may be asked to determine the uncertainty in the $y$-intercept by using the maximum and minimum lines and the line of best fit. As before, you can draw through points to obtain these lines or you can add error bars if you are comfortable doing so. Once the graph is plotted, with or without error bars, you need to do the following:

1. Draw the line of best fit and extrapolate it to find the 'best' value of the $y$-intercept.
2. Draw the maximum and minimum lines and extrapolate them back to get the maximum and minimum values for the $y$-intercept.
3. Determine the difference between the maximum and minimum values then halve it to find the uncertainty in the $y$-intercept value.
4. To find the percentage uncertainty in the $y$-intercept value, use the equation:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{(\text{'best' } y\text{-intercept value})} \times 100\%$$

**WORKED EXAMPLE 2**

A graph of potential difference against current gives a best fit line $y$-intercept value of 12.8 V, and maximum and minimum line intercept values of 11.4 V and 14.7 V respectively. Use this information to find:

(a) the uncertainty
(b) the % uncertainty in the $y$-intercept value
(c) the answer that should be stated.

**Answers**

(a) $\text{Uncertainty} = \tfrac{1}{2}(14.7 - 11.4)$

$= \tfrac{1}{2} \times 3.3 = \pm 1.65\,\text{V}$

(b) $\text{Percentage uncertainty} = \dfrac{1.65}{(12.8)} \times 100\%$

$= \pm 12.9\%$

(c) 12.8 ± 1.7 V or 12.8 ± 2 V

## Questions

By extrapolating the line of best fit, the minimum line and the maximum line to the $y$-axis in Figure 1, work out the % uncertainty in the $y$-intercept.

Copy the graph shown in Figure 2 and work out the % uncertainty in the gradient and in the $y$-intercept. The line of best fit has been drawn for you.

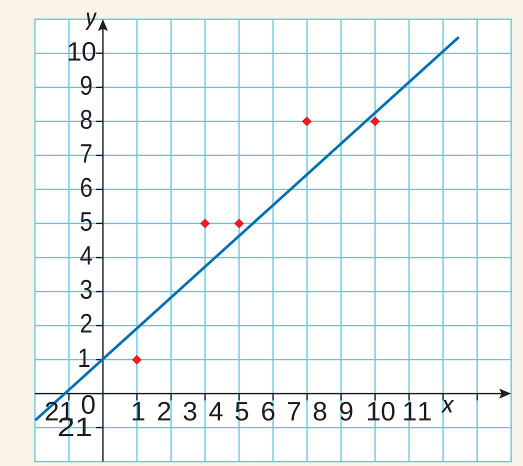



**Figure 2**

THINKING BIGGER

# ASSESSING A PRACTICAL WRITE-UP

Practical and investigative work is the lifeblood of physics. Without evidence to back up our theoretical claims, we can have no physical laws.

In this activity you will develop an understanding of how to improve practical techniques and the skills of implementing, planning, analysing and evaluating via the analysis of a sample of practical written work.

A student has been given the following task to plan implement, analyse and evaluate.

Task: Find the relationship between force and extension for a single spring, springs in series and springs in parallel.

**General introduction**

When a force of tension is applied to a spring, the extension will increase as the force increases in accordance with Hooke's law. The law states that the extension will be directly proportional to the load provided that the elastic limit is not exceeded.

Design and carry out a procedure to determine the spring constant for a single spring, springs in series and springs in parallel. Having done this, provide an explanation for how the value of the spring constant is different for the single spring, the series arrangement and the parallel arrangement. Your procedure should include:

1. A plan to include suitable apparatus and identification of variables to control.
2. A method which focuses on the practical techniques, suitable units and an appropriate format for the presentation of your data.
3. An analysis of the data, using correct equations, significant figures and values from graphs.
4. An evaluation of your procedure which includes a conclusion, a mention of accuracy and precision, anomalies in your data, percentage errors and any limitations in experimental procedures. You should also explain how to improve your procedures for future investigations.

**Student's write-up**

*Aim*

To find out about Hooke's law

*Method*

1. I set up the apparatus as shown in my diagram for a single spring.
2. I measured the length of the spring using a metre ruler that had cm rulings on it. I recorded this value.
3. I added masses to the spring and recorded the extension each time. I repeated this procedure to get two sets of data.
4. I did the same thing for two springs arranged in parallel, then I did it again for two springs in series.
5. I worked out the spring constants for each arrangement and compared them. My data is shown in Table 1.

*Calculations*

The single spring had a value for the spring constant of 4 N cm. The two springs in parallel had a spring constant of 8.0 N cm and the two springs in series had an effective spring constant of 2 Ncm. I plotted graphs of extension against force and found the gradient of each graph which gave me these values.

*Evaluation*

My values are accurate as they agree with Hooke's law. I could make my results more accurate by stating them to more decimal places and by taking more repeats. There are no anomalies in my data and they are all precise. I would guess that my values have a percentage error of about 10%.

1 Identify as many mistakes as you can in the planning and implementation stages of this investigation.

2 Comment on the data in Table 1. How is it presented well? How is it presented poorly?

| Single spring | | | Two springs in parallel | | | Two springs in series | | |
|---|---|---|---|---|---|---|---|---|
| Force/N | Extension/cm | | Force/N | Extension/cm | | Force/N | Extension/cm | |
| 1 | 4 | 4.0 | 1.0 | 8 | 8.0 | 2 | 4 | 4.0 |
| 3 | 12 | 12.2 | 2.0 | 16.5 | 17 | 4 | 8 | 9 |
| 4 | 15.5 | 16 | 3.0 | 25.5 | 26.0 | 6 | 12.0 | 12 |
| 7 | 28.8 | 29 | 4.0 | 31 | 41 | 8 | 17.5 | 18 |
| 11 | 44 | 43.5 | 5.0 | 40.00 | 40 | 10 | 20 | 23 |

**Table 1** The student's experimental results.

3 How accurate are the values stated in the conclusion?

4 Evaluate the student's evaluation of their own investigation.

> Look at the specification to check what is expected in order to produce a high-quality investigation write up. Familiarise yourself with the key elements of planning, designing, carrying out, analysing and evaluating a practical task.

**DID YOU KNOW?**

If you measure the time period of an oscillating spring, including two springs in series and two springs arranged in parallel, you can get an accurate value for the spring constant of the arrangement. This gives you another method for calculating, and verifying, the value obtained from the force–extension method.

## Activity

Re-write the student's write-up, so that the issues identified and addressed in your responses to the questions above are incorporated. Pay particular attention to the following key areas:

- Experimental design
- Identification of variables that need to be controlled and changed
- Use of a wide range of apparatus
- Appropriate units of measurement
- Appropriate presentation of data
- Use of appropriate mathematical skills
- Correct use of significant figures
- Plotting of graphs
- Measurement of gradient
- Accuracy of conclusions
- Percentage errors
- Identification of anomalies
- Suggesting improvements to the procedures, techniques and apparatus

# 2.1 Exam-style questions

1. Which of the following is greatest in value? [1]
   A Mega × nano
   B Mega ÷ nano
   C Giga ÷ micro
   D Kilo ÷ (milli)$^3$

**[Total: 1]**

2. Which quantity below has units that could be expressed as $kgms^{-2}$? [1]
   A Momentum
   B Energy
   C Impulse
   D Force

**[Total: 1]**

3. The radius of a metal sphere is 4.95 cm. What would be a good estimate of its volume? [1]
   A 80 $cm^3$
   B 500 $cm^3$
   C 1500 $cm^3$
   D 750 $cm^3$

**[Total: 1]**

4. Which of the following are the base units for the ohm? [1]
   A $kg\ m^3\ s^{-1}$
   B $kg\ m^2\ s^{-3}A^{-2}$
   C $kg\ m^{-1}A^{-1}$
   D $V\ A^{-1}$

**[Total: 1]**

5. Draw a line from the unit on the left-hand side to the equivalent unit on the right-hand side. [2]

| | |
|---|---|
| joule (J) | $kgms^{-2}$ |
| watt (W) | Nm |
| newton (N) | $Js^{-1}$ |

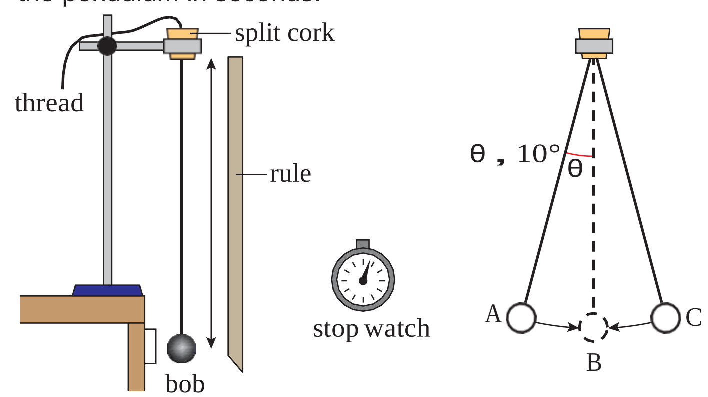
**[Total: 1]**

6. Match the prefix calculation on the left-hand side with the correct power on the right-hand side. [3]

| | |
|---|---|
| kilo × mega | $10^{-6}$ |
| kilo ÷ mega | $10^{9}$ |
| nano ÷ milli | $10^{-3}$ |
| micro × milli | $10^{-9}$ |

**[Total: 3]**

7. An experiment to determine the acceleration due to gravity, $g$, uses the equation $g = \frac{4pl^2}{T^2}$ where l is the length of the pendulum in metres and T is the time period of oscillation of the pendulum in seconds.




The readings taken were:
l = 1.250 ± 0.001 m and
T = 2.25 ± 0.02 s.

Calculate:

- the value of $g$ from this data [2]
- the percentage uncertainty in $g$. [2]

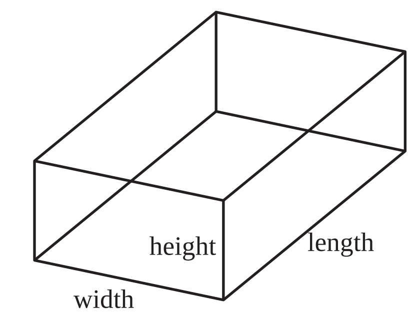
**[Total: 4]**

8. The force F acting on a body is obtained by using the equation $F = \frac{mv^2}{6(x_2 - x_1)}$. The values obtained in the investigation were $m = 50.0 \pm 0.5$ kg, $v = 6.0 \pm 0.2\ m\,s^{-1}$, $x_2 = 4.8 \pm 0.2$ m and $x_1 = 3.2 \pm 0.1$ m. Calculate:

- the size of the force, F, acting on the body in N [2]
- the percentage uncertainty in the value of F. [2]

**[Total: 4]**

9. An experiment is conducted to determine the density of a rectangular metal block. The mass of the block and its dimensions are as stated below:




- Length = 96 ± 0.5 mm
- Height = 15 ± 0.5 mm
- Width = 42 ± 0.5 mm
- Mass = 532 ± 0.5 g

The density calculation is performed twice – once with the actual numbers, and secondly with the highest possible numbers on the top of the equation and the lowest possible numbers on the bottom of the equation.

As density = $\frac{\text{mass}}{\text{volume}}$ show that the answer can be stated as

density = 8800 ± 500 kg. [5]

**[Total: 5]**